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# BRIEFCOMMUNICATION

## THE RELATIONSHIP BETWEEN DENSITY AND VOID FRACTION MEASUREMENT UNCERTAINTY IN RADIATION DENSITOMETRY†

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A review of the literature on use of radiation densitometry as a measurement technique in two-phase mixtures suggests some confusion about the relationship between the uncertainty estimates for void fraction and density. Several authors have derived static error expressions for radiation densitometer measurements. Honan & Lahey (1979) have developed an expression [1], which calculates the relative chordal void fraction error, $\Delta\alpha/\alpha$, as a function of chordal void fraction for a gamma ray densitometer system.

$$\frac{\Delta\alpha}{\alpha}=\frac{\sqrt{[2(1-a+\alpha^2)]}}{\alpha(\mu_L-\mu_G)L\sqrt{(\theta I_D A_D\epsilon)}} \qquad [1]$$

where

$$I_D=I_0\exp(-2\mu_W T)\exp\{[\mu_L+\alpha(\mu_G-\mu_L)]L\}.$$

and where $a$ is the chordal void fraction (time averaged), Stephens (1978) has derived an expression, [2], for the relative static density error, $\Delta\rho_{2\phi}/\rho_{2\phi}$, as a function of chordal density for a gamma ray densitometer system.

$$\frac{\Delta\rho_{2\phi}}{\rho_{2\phi}}=\frac{1}{kL\rho_{2\phi,(I\theta)}}. \qquad [2]$$

here $\rho_{2\phi}$ is the time averaged measured chordal two phase density and $k$ is the effective two-phase absorption coefficient $(m^{-1})$. $\mu$ are the macroscopic attenuation coefficient of the liquid phase $(L)$ and gas phase $(G)$, $I$ is the time averaged intensity (photon/s), $I_D$ is the intensity on the detector, $T$ is the test section wall thickness and, $L$ is the total path length through the two phase medium, $A_D$ is the area of beam incident on the detector, $\epsilon$ is the detector efficiency and $\theta$ is the counting interval (s). Subscript $W$ indicates wall. Martin (1972) has calculated a similar relative static chordal void fraction error for an X-ray system.

The relationship between relative static error in void fraction and density is obtained by a simple derivation. The two-phase chordal density can be written in terms of the void fraction as shown in [3]:

$$\rho_{2\phi}=\alpha\rho_G+(1-\alpha)\rho_L. \qquad [3]$$

The relationship between an uncertainty in density and the uncertainty in void fraction is

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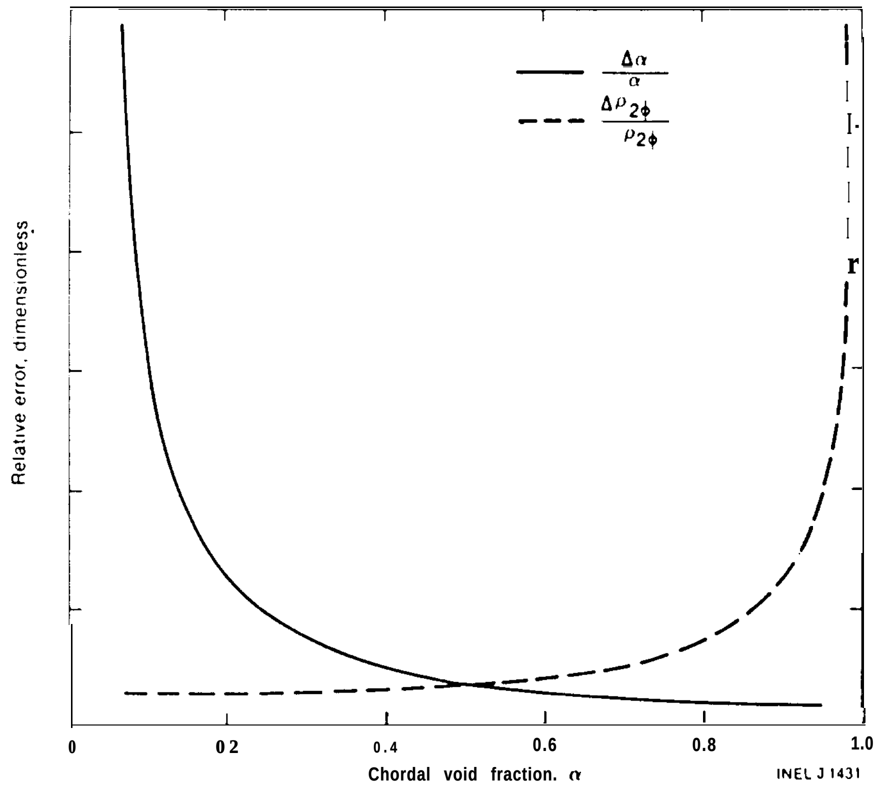


Figure I. Relative void fraction and density uncertainty as a function of void fraction.

obtained by taking the derivative of [3] with respect to a:

$$\Delta\rho_{2\phi} = -[\rho_L - \rho_G]\Delta\alpha. \qquad [4]$$

This expression yields the absolute error in void fraction, the constant of proportionality being the difference in the phase densities. Equation [4] can be divided by [3] and regrouped to yield

$$\frac{\Delta\rho_{2\phi}}{\rho_{2\phi}} = \frac{(\rho_L - \rho_G)}{(\rho_L - \rho_G) - \rho_L/\alpha}\frac{\Delta\alpha}{\alpha}. \qquad [5]$$

Note that the chordal void fraction appears in the expression that relates relative static density error to relative static void fraction error.

The relative static chordal void fraction error is calculated as a function of void fraction from [I] for a 6.68-cm pipe containing an air/water mixture at atmospheric pressure. Using [5], the relative chordal density error can be calculated as a function of chordal void fraction. These relative chordal void fraction and chordal density errors are shown graphically in figure I. Clearly a large relative chordal void fraction error does not imply a large relative density error.

Since density is proportional to liquid fraction, not void fraction, it follows that density and void fraction errors are inversely proportional. Figure 2 further illustrates the relationship between the relative static error in chordal void fraction and density for the same air/water mixture. Also shown in figure 2 is the relationship between absolute error in chordal void fraction and density, [4], for the same system. Note that the ratio of absolute density error to absolute void fraction is constant and fixed at the difference in density between the phases. The ratio of the relative errors is a strongly diverging void fraction.

An interesting aside is that Honan & Lahey's (1979) relative void fraction error expression is essentially identical to Stephen's (1978) relative density error. The Honan & Lahey expression

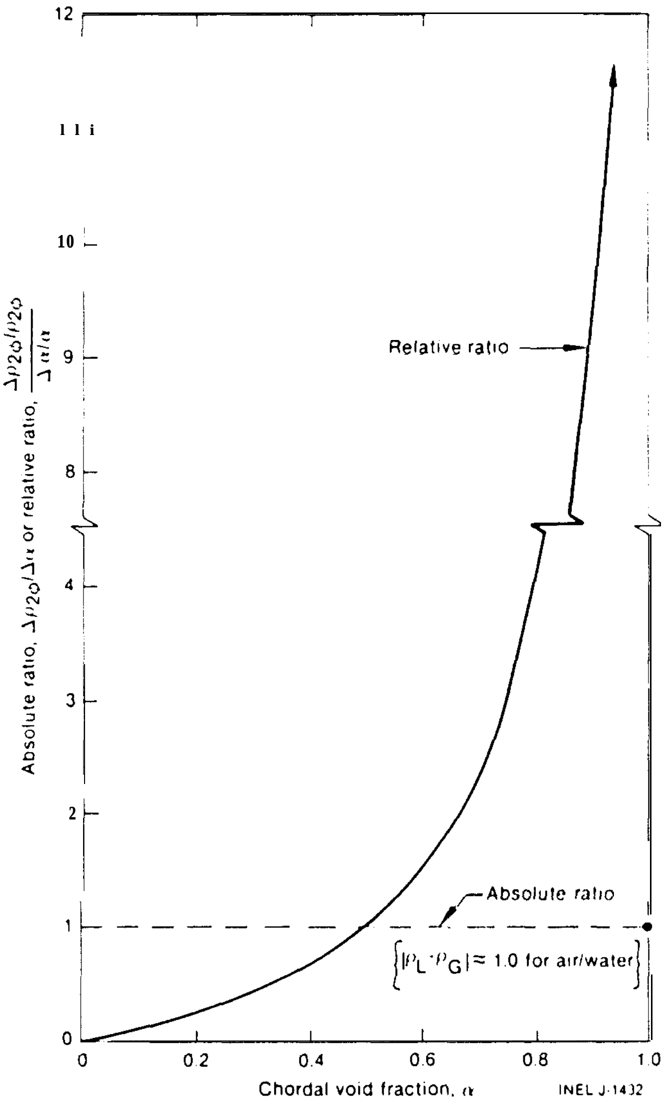


Figure 2. Relationship between relative and absolute chordal void fraction and chordal density uncertainty as a function of chordal void fraction.

contains $a\sqrt{[2(1-a+\alpha^2)]}$ term, which will not appear if long counting times are applied for the all vapor and all liquid calibration points. Equating Honan and Lahey's expression. [1], with the assumption that the reference counting times are long, to Stephens', [2], will produce [5].

In summary, the experimenter must be aware of the relationship between relative density uncertainty and relative void fraction uncertainty. The difference is very important when comparing various densitometer systems and data. The absolute chordal void fraction and the density errors are related by a multiplicative constant. In addition, the error expressions developed by two independent authors are essentially identical.

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